



# **Indicators of dispersion**

- If all observations are the same, there is *no variability*
- If they are not all the same, then dispersion is present in the data.
- *Variation* is an inherent characteristic of natural and experimental observations due to several reasons.
- It is always important to get an estimate of: (how much given objects tend to *differ from that central tendency*)
- In any experiment, variation will *depend on*:
  - > The instrument used for analysis.
  - $\blacktriangleright$  The analyst performing the assay.
  - > The sample chosen.
  - Unidentified error commonly known as.
- Why do we need indicators of dispersion?
  - A, B and C have the same mean (central tendency indicator).
  - Based on similarity of the mean, can we say the data sets are the same? No
  - > What is the difference between these data sets? Wideness of distribution (Variability).
  - ➢ How can we describe the (differences)? Using indicators of dispersion.

#### Indicators of dispersion:

- Standard deviation
- Coefficient of variation
- Variance
- Ouartiles
- Box and whisker plot
- Range (we don't use it because it is highly affected by extreme values/ outliers)

## **Standard deviation**

#### Lets start with an example:

- > We have batch of drug was compressed in 2 types of machines Alph and Bravo with a nominal content of 250mg then we took a random sample 500 tablets from each then we measured their drug content assays then we draw curve between (*drug content and frequency*)
- > The drug content distribution plots for Alpha & Bravo are normal distribution.
- What about the *central tendency*? Both have the same central tendency values *almost the same mean*













- Although tablets from both machines had equal mean, do you think the two machine still differ? How?
- What about *dispersion*, is it the same or different? *Alpha* has a <u>wider range</u> of tablet weights this means *higher variability*.
- Which of these 2 curves have fatter tails? *Alpha*, Fatter tail means tails with *higher frequency* of the extreme values.
- > Which of them has shorter middle value? *Alpha*

#### • So, we conclude that:

- Alpha tablet wights distribution has a <u>wider range and distribution</u>, <u>higher variability</u>, <u>fatter tails</u>, <u>shorter middle value</u>.
- The two machines are very similar in terms of average drug content for the tablets, both producing tablets with a mean very close to 250 mg. However, the two products clearly *differ*.
- With the *Alpha* machine, there is a considerable proportion of tablets with a content differing by more than 20 mg from the nominal dose (i.e. below 230 mg or above 270 mg).
- Whereas with the *Bravo* machine, <u>such outliers are much rarer</u>.
- An indicator of dispersion is required to convey this difference in variability and to decide which one has better performance!!

#### • Standard deviation formula:

$$\succ \text{ SD} = \sqrt{\frac{\sum_{i=1}^{n} (xi - \bar{x})^2}{n-1}}$$

- > This is the Standard deviation of a sample.
- $\blacktriangleright$  For a population it is donated as  $\sigma$

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (X-\mu)^2}{N}}$$

- > What is the unit of *SD* or  $\sigma$ ? The same unit of the observations.
- > Standard deviation is a widely used measure of variability and central dispersion
- Now let's go back to our *example* about the tableting machines:
  - We calculated the SD of the content of 10 tablets from an Alpha and Bravo tableting machines.
  - The Alpha machine produces rather variable tablets and so several of the tablets deviate considerably from the overall mean.
  - These relatively large figures then feed through the rest of the calculation, producing a high final SD (8.72 mg)
  - In contrast, the *Bravo* machine is more consistent and individual tablets never have a drug content much above or below the overall average.

		Alpha machine						
		Erythro	Deviation	$(X_{i} - X)^{2}$		Erythro	Deviation	
		content	from V - V	Deviation		content	from	Deviation
		(mg)	mean $\mathbf{A}_i = \mathbf{A}$	squared		(mg)	mean	squared
	ſ	249	0.3	0.09		251	-0.1	0.01
	×x, _	242	-6.7	44.89		247	-4.1	16.81
		252	3.3	10.89		257	5.9	34.81
		235	-13.7	187.69		250	-1.1	1.21
		257	8.3	68.89		254	2.9	8.41
		244	-4.7	22.09		251	-0.1	0.01
		264	15.3	234.09		252	0.9	0.81
		249	0.3	0.09		255	3.9	15.21
		255	6.3	39.69		244	-7.1	50.41
		240	<b>-8.</b> 7	75.69		250	-1.1	1.21
	(	Mean		Total		Mean		Total
	XX=	248.7		684.1		251.1		128.9
	Sum of squared deviations = 684.1 684.1/9 = 76.01 SD = square root 76.01				Sum of squared deviations = 128.9			
					128.9/9 = 14.32			
					SD = square root 14.32 = 3.78 mg (SD)			
	= 8.72  mg (SD)							

(more appropriate to place in the R & D section and more expensive).

> The small figures in the column of individual deviations, leading to a *lower SD* (3.78 mg)

#### • Reporting the SD:

- > The symbol  $\pm$  is used in reporting the SD
- > The symbol reasonably interpreted as meaning (more or less) is used to indicate variability.
- > With the tablets from our two machines, we would report their drug contents as:
  - ✓ *Alpha machine*: 248.7±8.72 mg (Mean ± SD mg)
  - ✓ *Bravo machine*: 251.1±3.78 mg (Mean ± SD mg)
- The two machines produce tablets with almost identical mean contents, but those from the *Alpha* machine are *two* to *three* times <u>more variable</u>.

### • Standard deviation and Coefficient of variation

- > Elephant tail = $150\pm10$  cm
- > Mouse tail= $7\pm3$  cm
  - Which is more variable: the elephant tail length results or the one for the mouse? *elephant tail length*
- CV=SD/Mean\*100
  - ✓ Elephant tail: CV=10/150\*100=**6.7%**
  - ✓ Mouse tail: CV= 3/7\*100=**42.8%** (*more variable*)
- > Coefficient of variation (CV) expresses variation relative to the magnitude of data.
- > Useful to compare variation in two or more sets of data with different mean values
- *CV has no unit* (it is a ratio).
- > A *higher* CV indicates greater variability in the data relative to the mean.
- A *lower* CV means less variability, suggesting that the data points are more closely clustered around the mean.
- ★ Example 1:
- Set A: 10, 20, 30, 40, 50 mean = 30
- Set B: 1, 2, 3, 4, 5 mean = 3
- > To know which set is more variable we calculate CV because the two sets have different means.

## ★ Example 2:

- ✓ 5 students got marks out of 100
  - **5**0, 60, 70, 80, 90
  - Mean = 70 & SD = 10
  - CV = 14.2%
- ✓ 5 students got marks out of 10
  - **5**, 6, 7, 8, 9
  - Mean = 7 & SD = 1
  - CV = 14.2%
- ✓ The have the same variability

## • Variance

- > The Variance:  $\sigma^2$  (population) or  $S^2$  (sample) is a measure of spread that is related to the deviations of the data values from their mean.
- > For population:  $\sigma^2 = \frac{\sum (X-\mu)^2}{N}$
- $\succ$  For sample:  $\mathbf{S}^2 = \frac{\sum (x \bar{x})^2}{n-1}$
- > Unit: <u>same as mean but squared</u> (If mean in mg, variance will be in  $mg^2$ )







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